

090107182029

ANNA UNIVERSITY OF TECHNOLOGY, COIMBATORE
B.E. / B.TECH. DEGREE EXAMINATIONS : NOV / DEC 2010

REGULATIONS : 2008

THIRD SEMESTER

080100008 - TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(COMMON TO CIVIL / EEE / EIE / ICE / ECE / BIOMEDICAL / BIOTECH /

AERO / AUTO / CSE / IT / MECHANICAL / CHEMICAL / FT / TT / TC)

Time: 3 Hours

Max.Marks: 100

PART - A

(20 x 2 = 40 Marks)

ANSWER ALL QUESTIONS

1. State the conditions for $f(x)$ to have Fourier series expansion.
2. Write a_0, a_n in the expansion of $x+x^3$ as Fourier Series in $(-\pi, \pi)$.
3. Expand $f(x)=1$ in a sine series in $0 < x < \pi$
4. Find Root Mean Square value of the function $f(x) = x$ in the interval $(0, l)$.
5. Define Fourier Transform Pair.
6. Find Fourier Cosine transform of e^{-2x} .
7. If $F(S)$ is the Fourier Transform of $f(x)$, show that the Fourier Transform of $e^{iax} f(x)$ is $F(S+a)$.
8. State Parseval's Identity for Fourier Transform.
9. Eliminate the arbitrary constants a & b from $z = (x^2 + a)(y^2 + b)$.
10. Form the PDE by eliminating the functions from $z = f(x+t) + g(x-t)$.
11. Find the complete integral $q = 2px$.
12. Solve $(D^3 - 3DD'^2 + 2D'^3)z = 0$.

13. Find the nature of PDE $4u_{xx} + 4u_{xy} + u_{yy} + 2u_x - u_y = 0$.
14. What are the various solutions of one dimensional Wave Equation?
15. A string is stretched and fastened to two points 'l' apart. Motion is started by displacing the string into the form $y = y_0 \sin(\frac{\pi x}{l})$ from which it is released at time $t=0$.
Formulate this problem as the boundary value problem.
16. A rod of length 20cm whose one end is kept at 30°C and the other end is kept at 70°C is maintained so until steady state prevails. Find the steady state temperature.
17. Find $Z[e^{-an}]$.
18. Prove that $Z[n] = \frac{z}{(z-1)^2}$
19. Prove that $Z[f(n+1)] = zF(z) - zf(0)$
20. State Initial and Final value theorem on Z- transform.

PART - B

(5 x 12 = 60 Marks)

ANSWER ANY FIVE QUESTIONS

21(a). If $f(x) = \left(\frac{\pi-x}{2}\right)$ find the Fourier Series of the period 2π in the interval $(0, 2\pi)$.

Hence deduce that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots = \frac{\pi}{4}$ (8)

(b). Find the Fourier expansion of $f(x) = x$ in the interval $(-\pi, \pi)$ (4)

22. Show that the Fourier Transform of $f(x) = \begin{cases} a^2 - x^2 & |x| \leq a \\ 0 & \text{otherwise} \end{cases}$ is

$$2\sqrt{\frac{2}{\pi}} \left(\frac{\sin as - as \cos as}{s^3} \right) \text{ Hence deduce that } \int_0^{\infty} \frac{\sin t - t \cos t}{t^3} dt = \frac{\pi}{4}$$

Using Parseval's Identity show that $\int_0^{\infty} \left(\frac{\sin t - t \cos t}{t^3} \right)^2 dt = \frac{\pi}{15}$

23.(a) Solve $(mz-ny)p + (nx-lz)q = ly-mx$ (6)

(b) Solve $(D^3 + D^2D' - DD'^2 - D'^3)z = e^x \cos 2y$ (6)

24. A string of length l is initially at rest in its equilibrium position and motion is started by giving each of its points a velocity $V = \begin{cases} cx & , 0 \leq x \leq l/2 \\ c(l-x), & l/2 \leq x \leq l \end{cases}$
Find the displacement function $y(x,t)$.

25 (a) Evaluate $z^{-1} \left[\frac{z}{z^2 + 7z + 10} \right]$ (6)

(b) Using z-transforms solve $u(n+2) - 5u(n+1) + 6u(n) = 4^n$ given that $u(0)=0, u(1)=1$ (6)

26(a) Find the constant term and the coefficient of the first sine and cosine terms in the Fourier expansion of, $y=f(x)$ as given in the following table:- (6)

| | | | | | | | |
|------|---|----|----|----|----|----|---|
| x | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| f(x) | 9 | 18 | 24 | 28 | 26 | 20 | 9 |

26(b) Find the Fourier transform of $f(x) = \begin{cases} 1-|x| & |x| \leq 1 \\ 0 & \text{otherwise} \end{cases}$

hence find the value of $\int_0^{\infty} \frac{\sin^4 x}{x^4} dx$ (6)

27. A metal bar 30cm long has its ends A and B kept at 20°C and 80°C respectively, until steady state conditions prevail. The temperature at each end is then suddenly reduced to 0°C and kept so. Find the resulting temperature $u(x,t)$ taking $x=0$ at A.

28(a) Solve $p(1+q) = qz$ (6)

(b) Using Convolution theorem, evaluate $z^{-1} \left[\frac{z^2}{(z-1)(z-3)} \right]$ (6)

*****THE END*****