

Madras University

M.C.A Introduction to Discrete Structures Question paper

Time: Three hours

Maximum: 75 marks

PART A - [5 x 5 = Marks 25]

1. (a) Show the equivalence
 $(P \wedge Q) \vee (P \wedge \neg Q) \vee (P \wedge Q)$

Or

(b) Show the $\{\neg, \vee\}$ and $\{\vee\}$ are not functionally complete.

2. (a) Let Z be the set of integers and let R be the relation called "congruence modulo 3" defined by $R = \{x, y \mid x \equiv y \pmod{3} \text{ (i.e., } x - y \text{ is divisible by 3)}\}$. Determine the equivalence classes generated by the elements of Z .

Or

(b) Give an example of a set X such that (X, \leq) is a totally ordered set.

3. (a) State and prove Lagrange's theorem.

Or

(b) Prove that every subgroup of a cyclic group is cyclic.

4. (a) Explain the method of finding the adjacency matrix.

Or

(b) How will you represent a graph using arrays.

5. (a) Explain the term 'right derivation' using a grammar to generate a language.

Or

(b) Give a context-free grammar which generates $L = \{w \mid w \text{ contains twice as many 0s as 1s}\}$

PART B - [5 x 10 = Marks 50]

Answer any FIVE questions.

All questions carry equal marks.

6. Show that $\forall x (P(x) \vee Q(x)) \rightarrow \exists x (P(x) \vee (\exists x) Q(x))$.

7. show that $\exists x (P \vee Q)$ is a valid conclusion from the premises $P \vee Q$, $Q \rightarrow R$, $P \rightarrow M$ and M .

8. Show that addition of matrices form an abelian group.

9. Prove that for any commutative monoid $M, *$, the set of idempotent elements of M forms a submonoid.

10. Show that for any two sets A and B $A - (A \cap B) = A - B$.

11. (a) Show that the set of divisors of a positive integer n is recursive.

(b) Show that the sets of even and odd natural numbers are both recursive.

12. Contrast a context-sensitive grammar for the language $\{ w \mid w \in \{a,b,c\}^* \}$, where w contains the same number of a's, b's and c's.

13. Explain the terms 'sentential form', regular grammar and grammars.