Reg. No. :

Question Paper Code : 21527

B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2013.

Fifth Semester

Computer Science and Engineering

MA 2265/MA 52/10144 CS 501 — DISCRETE MATHEMATICS

(Regulation 2008/2010)

(Common to PTMA 2265 – Discrete Mathematics for B.E. (Part-Time) Third Semester – Computer Science and Engineering – Regulation 2009)

Time : Three hours   Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

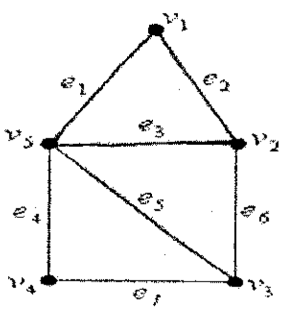
1. What are the contrapositive, the converse, and the inverse of the conditional statement. "*If you work hard then you will be rewarded.*"

2. Find the truth table for the statement $P \to Q$.

3. In how many ways can all the letters in *MATHEMATICAL* be arranged.

4. Twelve students want to place order of different ice-creams in a ice- cream parlour, which has six type of ice-creams. Find the number of orders that the twelve students can place.

5. Obtain the adjacency matrix of the graph given below.



6. Give an example of a non-Eulerian graph which is Hamiltonian.

7. Prove that if G is abelian group, then for all $a,b \in G$ $(a*b)^2 = a^2 * b^2$.

8. Show that every cyclic group is abelian.

9. Show that least upper bound of a subset B in a poset $(A,\leq)$ is unique if it exists.

10. Given an example of a distributive lattice but not complemented.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Prove that $\sqrt{2}$ is irrational by giving a proof by contradiction. (8)

(ii) Show that (8)

$$(\neg P \to R) \wedge (Q \leftrightarrow P) = (P \vee Q \vee R) \wedge (P \vee \neg Q \vee R) \wedge (P \vee \neg Q \vee \neg R) \wedge (\neg P \vee Q \vee R) \wedge (\neg P \vee Q \vee \neg R)$$

Or

(b) (i) Prove that $(\forall x)(p(x) \vee q(x)) \Rightarrow (\forall x)p(x) \vee (\exists x)\, q(x)$.

(ii) Prove that $(P \to Q) \wedge (R \to Q) \Rightarrow (P \vee R) \to Q$.

12. (a) (i) Using generating function, solve the recurrence relation $a_n - 5a_{n-1} + 6a_{n-2} = 0$ where $n \geq 2$, $a_0 = 0$ and $a_1 = 1$. (10)

(ii) Let m any odd positive integer. Then prove that there exists a positive integer n such that m divides $2^n - 1$. (6)

Or

(b) (i) Determine the number of positive integers n, $1 \leq n \leq 2000$ that are not divisible by 2, 3, or 5 but are divisible by 7. (10)

(ii) State the Strong Induction (the second principle of mathematical induction). Prove that a positive integer greater than 1 is either a prime number or it can be written as product of prime numbers. (6)

13. (a) (i) Prove that if G is a simple graph with at least three vertices and $\delta(G) \geq \frac{|V(G)|}{2}$ then G is Hamiltonian. (10)

(ii) Check whether the two graphs given in Figure Q 13(a) are isomorphic or not. (6)

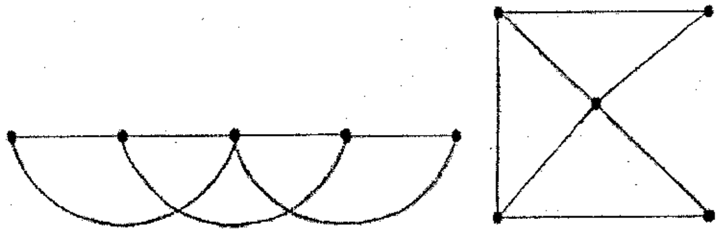
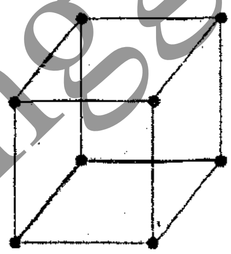


Figure. Q(13)

Or

(b). (i) Let G be a simple undirected graph with adjacency matrix A with respect to the ordering $v_1, v_2, v_3, \ldots, v_n$. Prove that the number of different walks of length r from v_i to v_j equals the (i, j) th entry of A^r, where r is a positive integer. (8)

(ii) Check whether the graph given below is Hamiltonian or Eulerian or 2- colorable. Justify your answer. (4)



(iii) Show that if a graph with n vertices is self-complementary then $n \equiv 0$ or $1 \pmod 4$. (4)

14. (a) (i) Prove that in a finite group, order of any subgroup divides the order of the group. (10)

(ii) Prove that intersection of two normal subgroups of a group $(G,*)$ is a normal subgroup of a group $(G,*)$. (6)

Or

(b) (i) Prove that every finite group of order n is isomorphic to a permutation group of degree n. (10)

(ii) Let $(G,*)$ and (H, Δ) be two groups and $g:(G,*) \to (H,\Delta)$ be group homomorphism. Then prove that the Kernel of g is normal subgroup of $(G,*)$. (6)

15. (a) (i) Let L be lattice, where

$a*b = \text{glb}(a,b)$ and $a \oplus b = \text{lub}(a,b)$ for all $a,b \in L$. Then both binary operations $*$ and $\oplus$ defined as in L satisfies commutative law, associative law, absorption law and idempotent law. (8)

(ii) Show that in a distributive and complemented lattice satisfied De Morgan's laws. (8)

Or

(b) (i) Show that every chain is a lattice. (8)

(ii) Show that in a distributive and complemented lattice

$a \le b \Leftrightarrow a*b' = 0 \Leftrightarrow a' \oplus b = 1 \Leftrightarrow b' \le a'$. (8)
