Reg. No. : [ ][ ][ ][ ][ ][ ][ ][ ][ ][ ][ ]

Question Paper Code : Q 2722

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2009.

First Year — Annual Pattern

Computer Science and Engineering

MA 1X01 — ENGINEERING MATHEMATICS — I

(Common to all branches)

(Regulation 2004)

Time : Three hours   Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. If 0.5, 1.5 are eigenvalues of $A = \begin{pmatrix} 1 & x \\ 1 & 1 \end{pmatrix}$, find the value of x.

2. Find the curvature of the curve $x^2 + y^2 + 4x - 1 = 0$.

3. Reduce $x^2 \frac{d^2y}{dx^2} + 3y = 0$ into a differential equation with constant coefficients.

4. Find the particular integral of $\frac{d^2y}{dx^2} - y = 2^x$.

5. Change the order of integration in $\int_0^1 \int_x^1 \frac{1}{x^2 + y^2} dx\, dy$.

6. Find the tangent plane at (6, 4, 3) to $xy + yz + zx = 54$.

7. Can $x^2 + y^2 - 2xy$ be the real part of an analytic function? Justify your answer.

8. Find the residue at $z = 0$ of $f(z) = \dfrac{1}{z^2(z+1)}$.

9. Verify the initial value theorem given $f(t) = $ 't'.

10. What is the inverse Laplace transform of $\dfrac{1}{(s+2)^2}$?

PART B — ($5 \times 16 = 80$ marks)

11. (a) (i) Show that the line joining any point θ on $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$ to its centre of curvature is bisected by the line $y = 2a$. (10)

(ii) Find the inverse of $A = \begin{pmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{pmatrix}$ using Cayley Hamilton theorem. (6)

Or

(b) (i) Reduce the quadratic form $-x^2 + y^2 + 4yz + 4zx$ into canonical form. (8)

(ii) Find the maxima and minima of $f(x, y) = \sin x \sin y \sin(x + y)$ given $0 < x, y < \pi$. (8)

12. (a) (i) Solve $\dfrac{d^2y}{dx^2} + 121y = \tan 11x$ by the method of variation of parameters. (8)

(ii) Determine the bending curve $y(x)$ where $y(x)$ satisfies the differential equation $\dfrac{d^2y}{dx^2} = \dfrac{P(l - x)}{EI}$, given $y = 0$ and $\dfrac{dy}{dx} = 0$ at $x = 0$. (8)

Or

(b) (i) Solve $(D^2 + 2D + 1)y = 2e^{-x} + \cos x + x^2 + 1$. (8)

(ii) A particle is executing a simple harmonic motion $\dfrac{d^2x}{dt^2} = -\mu x$. At $t = 0$, $x = a$ and velocity $v = 0$. Find the time taken to go from the position $x = \dfrac{a}{2}$ to $x = a$. Also prove that this time is $\dfrac{1}{6}$ of the period. (8)

13. (a) (i) Evaluate $\iiint dx\,dy\,dz$ over the volume cut off the sphere $x^2+y^2+z^2=a^2$ by the cone $x^2+y^2=z^2$. (8)

(ii) A vector field is given by $F=(x^2-y^2+x)i-(2xy+y)j$. Show that the field is irrotational and find its scalar potential. Hence, evaluate the line integral from (1, 2) to (2, 1). (8)

Or

(b) (i) Find a and b so that the surfaces $ax^2-byz=(a+2)x$ is orthogonal to $4x^2y+z^3=4$ at (1, –1, 2). (6)

(ii) Verify Stoke's theorem for $F=(y-z)i+yz\,j-xz\,k$ where S is the surface bounded by the planes $x=0$, $x=1$, $y=0$, $y=1$, $z=0$ and $z=1$ above the xoy plane. (10)

14. (a) (i) If $u=x^2-y^2$ and $v=-\dfrac{y}{x^2+y^2}$, prove that both u and v satisfy Laplace equation, but that $u+iv$ is not a regular function of z. (8)

(ii) Using Cauchy's integral formula for derivatives, evaluate $\int_C \dfrac{e^{2z}}{(z+1)^4}dz$ where C is the circle $|z|=2$. (8)

Or

(b) (i) Evaluate $\int_0^{2\pi}\dfrac{d\theta}{1-2a\cos\theta+a^2}$ when $|a|<1$. (8)

(ii) Find the bilinear transformation if 1 and i are fixed points and the origin in z-plane goes to –1 in w-plane. (8)

15. (a) (i) Solve $ty'-y=5$ by using Laplace Transform given $y(1)=10$. (8)

(ii) Using convolution theorem find the inverse of $\dfrac{s}{(s^2+121)^2}$. (8)

Or

(b) (i) Find the Laplace transform of the square wave function

$$f(t) = \begin{cases} k \text{ in } 0 \le t \le a \\ -k \text{ in } a \le t \le 2a \end{cases} \text{ and } f(t+2a) = f(t) \text{ for all } t.$$ (6)

(ii) Solve the simultaneous equations :

$D^2x - Dy = \cos t$ and

$Dx + D^2y = -\sin t$ where $D = \frac{d}{dt}$

given that $x = 1$, $Dx = 0$, $y = 0$ and $Dy = 1$ at $t = 0$. (10)
