Reg. No. : [ ][ ][ ][ ][ ][ ][ ]

Question Paper Code : 11016

B.E./B.Tech. DEGREE EXAMINATION, JUNE 2011.

Common to all B.E./B.Tech.

First Semester

181101 — MATHEMATICS — I

(Regulation 2010)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. If A and B are non-Singular matrices then prove that AB and BA will have the same eigenvalues.

2. If the matrix A is given by

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2+i & -1 & 0 & 0 \\ -3 & 2i & i & 0 \\ 4 & -i & 1 & -i \end{bmatrix}$$ where $i = \sqrt{-1}$, then using cayley hamilton theorem prove that $A^4 = I$.

3. Find the equation of the sphere with centre at (2,3,5) which touches XOY-plane.

4. Find the equation of a Cone whose vertex is at the origin and the guiding curve is $\frac{x^4}{4} + \frac{y^2}{9} + z^2 = 1, x + y + z = 1$.

5. Find the curvature of the curve $x^2 + y^2 + 2x + 2y - 1 = 0$ at any general point (x,y).

6. Find the envelope of $x - y\sin\theta = a\cos\theta$ where θ is the parameter.

7. If $u = xy$ and $v = x + y$ find $\frac{\partial(x, y)}{\partial(u, v)}$.

8. Find the Taylor series expansion of $e^x \sin y$ near the point $(-1, \frac{\pi}{4})$ upto the second degree terms.

9. Evaluate $\int_a^b \int_c^d \int_f^g e^{x+y+z} dz\, dy\, dx$.

10. Change the order of integration in $\int_0^\infty \int_x^\infty f(x, y)\, dy\, dx$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the eigenvalues and eigenvectors of the matrix A^{-1} given $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ (8)

(ii) Diagonalise the matrix $A = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ by means of an orthogonal transformation. (8)

Or

(b) (i) Using Caylay Hamilton theorem evaluate the matrix

$A^4 + A^3 - 18A^2 - 39A + 2I$, given the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix}$ (8)

(ii) Reduce the Quadratic Form $x^2 + y^2 + z^2 - 2xy - 2yz - 2zx$ into a Canonical form through an orthogonal transformation. (8)

12. (a) (i) Find the equation of the tangent planes to the sphere $x^2 + y^2 + z^2 - 4x - 2y - 6z + 5 = 0$ which are parallel to the plane $x + 4y + 8z = 0$. (8)

(ii) Show that the equation of the right circular cone with its vetex at (0,0,0) with Z-axis as its axis and the semivertical angle equal to α is $x^2 + y^2 = z^2 \tan^2 \alpha$. (8)

Or

(b) (i) Find the equation of the sphere for which the circle $2x + 3y + 4z = 8, x^2 + y^2 + z^2 + 7y - 2z + 2 = 0$ is a great circle. (8)

(ii) Find the equation of the right circular cylinder of radius 3 units whose axis passes through (2,3,4) and has direction cosines proportional to 2, 1, −2. (8)

13. (a) (i) Show that the curves $y = \frac{a}{2}(e^{\frac{x}{a}} + e^{\frac{-x}{a}})$ and $y = \frac{a}{2}(2 + \frac{x^2}{a^2})$ have the same curvature at their crossing with Y-axis. (8)

(ii) Circles are drawn with centres on the ellipse. Find the envelope of all such circles which pass through the origin. (8)

Or

(b) (i) Find the evaluate of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. (8)

(ii) Find the envelope of $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ where the parameters a and b are connected by the reltion $ab = k^2$. (8)

14. (a) (i) If $g(x, y) = \psi(u, v)$ where $u = x^2 - y^2, v = 2xy$, prove that $\frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2} = 4(x^2 + y^2)\left[\frac{\partial^2 \psi}{\partial u^2} + \frac{\partial^2 \psi}{\partial v^2}\right]$. (8)

(ii) Find the shortest and longest distances from the point (1, 2, −1) to the sphere $x^2 + y^2 + z^2 = 24$, using the method of Lagrange multipliers. (8)

Or

(b) (i) Expand $e^{-x} \log y$ as a Taylor series in powers of x and $(y-1)$ upto third degree terms. (8)

(ii) Find the Jacobian of y_1, y_2, y_3 with respect to x_1, x_2, x_3 if $y_1 = \frac{x_2 x_3}{x_1}$, $y_2 = \frac{x_3 x_1}{x_2}$ and $y_3 = \frac{x_1 x_2}{x_3}$. (8)

15. (a) (i) Evaluate by changing the order of integration in $\int_0^a \int_y^a \frac{x}{x^2 + y^2} \, dx \, dy$. (8)

(ii) Find by triple integration the volume of the Ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$. (8)

Or

(b) (i) Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} \, dx \, dy$ by transforming into polar co-ordinates. (8)

(ii) Find the volume bounded by the cylinder $x^2 + y^2 = 4$ and the planes $y + z = 4$ and $z = 0$. (8)
